



# Damping of Quasi-Stationary Waves Between Two Miscible Liquids

Walter M.B. Duval  
Glenn Research Center, Cleveland, Ohio

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Walter M.B. Duval  
Glenn Research Center, Cleveland, Ohio

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# DAMPING OF QUASI-STATIONARY WAVES BETWEEN TWO MISCIBLE LIQUIDS

Walter M.B. Duval  
National Aeronautics and Space Administration  
Glenn Research Center  
Cleveland, Ohio 44135

Two viscous miscible liquids with an initially sharp interface oriented vertically inside a cavity become unstable against oscillatory external forcing due to Kelvin-Helmholtz instability. The instability causes growth of quasi-stationary (q-s) waves at the interface between the two liquids. We examine computationally the dynamics of a four-mode q-s wave, for a fixed energy input, when one of the components of the external forcing is suddenly ceased. The external forcing consists of a steady and oscillatory component as realizable in a microgravity environment. Results show that when there is a jump discontinuity in the oscillatory excitation that produced the four-mode q-s wave, the interface does not return to its equilibrium position, the structure of the q-s wave remains imbedded between the two fluids over a long time scale. The damping characteristics of the q-s wave from the time history of the velocity field show overdamped and critically damped response; there is no underdamped oscillation as the flow field approaches steady state. Viscous effects serve as a dissipative mechanism to effectively damp the system. The stability of the four-mode q-s wave is dependent on both a geometric length scale as well as the level of background steady acceleration.

## Introduction

Quasi-stationary waves generated at the interface between two miscible liquids are important for understanding transport processes in a microgravity environment. These quasi-stationary (q-s) waves, generated via a controlled vibration source, serve as model to understand effects of g-jitter. In the course of an experiment one is interested in knowing how long it takes the injected energy from the vibration source to decay once it is ceased. The decay time interval is important to experimentalists, since it serves as a marker to signal when change can be made to the input amplitude and frequency of excitation to the experiment.<sup>1</sup> To obtain insight into the problem, we consider the damping characteristic of a q-s wave due to jump discontinuity of the oscillating component of the body force after a finite growth time interval.

The characteristic feature of q-s waves inside a bounded enclosure is that they are nonpropagating internal waves of permanent form between a density interface, generated via an oscillating parallel shear flow through coupling with the body force. A continuous external excitation is maintained during growth of q-s waves. Unlike internal waves of permanent form inside

heterogeneous fluids, solitary and periodic cnoidal waves,<sup>2-6</sup> q-s waves do not propagate; thus their phase velocity is zero. Nonpropagating solitary waves<sup>7,8</sup> have been generated inside narrow channels partially filled with water undergoing continuous excitation, in contrast to q-s waves there is a free surface. Another feature of q-s waves is that they have long wavelengths similar to gravity waves occurring on free surfaces.

Owing to viscous dissipation between the two fluids, damping from external excitation is very effective for q-s waves in comparison for liquids inside containers with a free surface.<sup>9,10</sup> As pointed out by Chandrasekar,<sup>11</sup> the problem of gravity waves which occur in an infinitely deep ocean or the surface of a fluid with a free surface is an analogue to the problem of oscillation of a viscous drop, other than its spherical geometry; for gravity waves the dispersion relation is a special condition of the more general Rayleigh-Taylor instability problem of superposed fluids with a jump in density and viscosity across a density interface. Since we are interested on the effects of microgravity, there has been experimental<sup>12,13</sup> and theoretical studies<sup>14-16</sup> on damping of viscous drops either with a free surface or immersed in another fluid in microgravity. These studies consider the damping

characteristics of a liquid drop when the excitation force is suddenly ceased. The results show that oscillations decay either periodically, with decreasing amplitude similar to an underdamped harmonic oscillator, or aperiodically depending on the parametric region.

The damping characteristic of a q-s wave is addressed computationally. We show that for a body force consisting of steady and oscillatory component, the q-s wave exhibits either an overdamped or critically damped response once the oscillatory component ceases. This behavior is similar to a damped harmonic oscillator, with the exception that no underdamped response is predicted as found for viscous liquid drops or plane liquid surface inside an enclosure with a free surface.<sup>9,12,13</sup> The effect of length scale on the characteristics of q-s wave is also investigated. We show that the existence of a four-mode q-s wave has both a geometrical and background acceleration level restriction. Thus the parametric space is not arbitrary but has to be chosen with care.

In the following, we discuss the model problem and relevant length scale. The damping characteristic of the q-s wave is deduced from its structure and time history of its velocity field. We extract the geometrical and background acceleration level for the existence of the four-mode q-s wave from its local bifurcation as the length scale varies as well as its background acceleration level.

### Formulation

The realization shown in Figure 1, of two miscible liquids oriented vertically inside a cavity with an initial jump in density across its interface, has been achieved in a microgravity environment.<sup>1</sup> The equilibrium condition of a stable interface requires that the background steady acceleration ( $ng_o$ ) be on the order of  $1-\mu g$ , while the unsteady component ( $mg_o$ ) is zero. This implies that over a long time scale the interface obeys the linear diffusion equation,

$$\frac{\partial C}{\partial t} = D_{ab} \nabla^2 C \quad (1)$$

When energy is injected into the system with input amplitude ( $mg_o$ ), circular frequency  $\omega$ , and phase angle  $\theta$ , the interface will become unstable and generate a q-s wave. The mechanism that leads to growth of a q-s wave is an oscillating parallel shear flow, which generates a line vortex due to tangential discontinuity of the velocity field near the interface.<sup>17</sup> The interface becomes unstable

against Kelvin-Helmholtz instability and growth of a q-s wave occurs. We are interested on the damping characteristics of the q-s wave when the energy input into the system ceases. This can be modeled by a jump discontinuity in the body force such that,

$$\vec{g}(t) = \begin{cases} ng_o + mg_o(\cos(\omega t + \theta)) & t < t_c \\ ng_o & t \geq t_c \end{cases} \quad (2)$$

where  $t_c$  is the cut-off time of energy injection. For  $t < t_c$  energy injection into the system leads to non-equilibrium which must satisfy the Boussinesq momentum equation,

$$\bar{\rho} \frac{D\vec{V}}{Dt} = -\nabla p + \bar{\mu} \nabla^2 \vec{V} + \rho \vec{g}(t) \quad (3)$$

as well as the incompressibility condition,

$$\nabla \cdot \vec{V} = 0 \quad (4)$$

The density field is a linear function of concentration,

$$\rho = \bar{\rho}(1 + \beta \Delta C) \quad (5)$$

Since the flow field becomes intense, convective acceleration must be taken into account in the species continuity equation,

$$\frac{DC}{Dt} = D_{ab} \nabla^2 C \quad (6)$$

The interface is prescribed by the following initial condition,

$$C(x, y, 0) = \begin{cases} 1.0 & 0 \leq x \leq L/2 \\ 0.5 & x = L/2 \\ 0 & L/2 \leq x \leq L \end{cases} \quad (7)$$

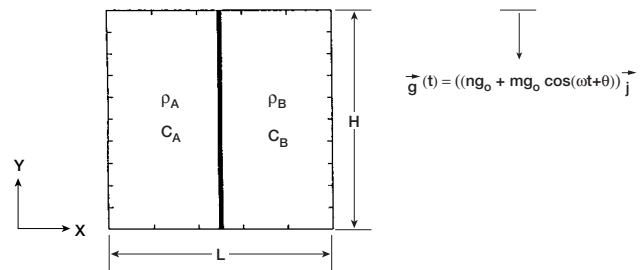


Figure 1.—Initial condition of the interface between two miscible liquids.

The interface has the value of 0.5, while the left and right fluid has values of 1 and 0, respectively; the gradient at the interface lies between .9 to 0.1. Overbar in the above equations denotes average values. The boundary conditions at the walls are no-slip along the wall boundaries  $\Gamma$ ,

$$\vec{V} \bullet \vec{n} = 0 \text{ on } \Gamma \quad (8)$$

and the condition of impermeability normal  $\vec{n}$  to the boundary,

$$\nabla C \bullet \vec{n} = 0 \text{ on } \Gamma \quad (9)$$

Equations (3,4,5) with its initial (7) and boundary conditions (8,9) allows the prediction of the flow field  $\vec{V}(x, y, t)$  as well as its wavelength  $\lambda(x, y, t)$  as a function of a parameter space  $\Lambda$ . The parameter space may be deduced taking the curl of equation (3) and non-dimensionalizing with the appropriate length and time scale<sup>18</sup>. The vorticity field equation in its two-dimensional form can be expressed as,

$$\frac{D\xi}{Dt} = \bar{v} \nabla^2 \xi + (ng_o + mg_o \cos(\omega t + \theta)) \frac{\partial C}{\partial x} \quad (10)$$

While the continuity equation (4) is transformed to a Poisson's equation,

$$\nabla^2 \psi = -\xi \quad (11)$$

The velocity components and vorticity are defined as,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

For a characteristic length ( $\mathcal{L}=H$ ), time ( $T=1/\omega$ ), and velocity ( $U_c = \Delta\rho / \bar{\rho} \bullet mg_o H^2 / \bar{v}$ ) scale, the parametric space can be expressed as

$$\Lambda_H = \Lambda_H(Ar, Gr_m / Re, 1/Re, AR, Sc, \theta) \quad (12)$$

The parametric space consisting of six parameters is the aspect ratio ( $Ar$ ), the Grashof number ( $Gr_m$ ), Reynolds number ( $Re$ ), amplitude ratio ( $AR$ ), Schmidt number ( $Sc$ ), and the phase angle ( $\theta$ ),

$$Ar = \frac{H}{L}, \quad Gr_m = \frac{\Delta\rho}{\bar{\rho}} \frac{mg_o H^3}{\bar{v}^2}, \quad AR = \frac{n}{m},$$

$$Re = \frac{\omega H^2}{\bar{v}}, \quad Sc = \frac{\bar{v}}{D_{ab}}, \quad Gr_m / Re = \frac{\Delta\rho}{\bar{\rho}} \frac{mg_o H}{\omega \bar{v}}$$

The ratio of  $Gr_m/Re$  dictates the importance of nonlinearity in the field equations. The amplitude ratio is also the ratio of Grashof numbers ( $AR=Gr_n/Gr_m$ ) based on  $ng_o$  and  $mg_o$  respectively. For a Stokes-length scale ( $\mathcal{L}=\delta$ ) based on diffusion of momentum ( $\delta = \sqrt{\bar{v}/\omega}$ ), the parametric space is simplified and reduces to four parameters

$$\Lambda_\delta = \Lambda_\delta(Re s, AR, Sc, \theta) \quad (13)$$

These parameters are essentially independent of geometric length scale; the Stokes-Reynolds ( $Re s$ ) number is defined as

$$Re s = \frac{\Delta\rho}{\bar{\rho}} \frac{mg_o}{\omega^{3/2} \bar{v}^{1/2}}$$

For zero phase angle and fixed fluid properties ( $Sc=1079$ ),  $Sc$  and  $\theta$  are essentially passive parameters, thus  $\Lambda$  based on geometric and boundary layer length scale reduces respectively to

$$\Lambda_H = \Lambda_H(Ar, Gr_m / Re, 1/Re, AR)$$

$$\Lambda_\delta = \Lambda_\delta(Re s, AR)$$

Use of the boundary-layer length scale reduces the parametric space to a co-dimension two bifurcation problem. Of interest is the region of applicability of the boundary layer versus the geometric length scale in parametric space. This will be deduced from the bifurcation of the wavelength of the interface,

$$\lambda = \lambda(x, y, t; Re s, AR)$$

using a known experimental condition as reference.<sup>1</sup> The velocity field can also be expressed by the same functional relationship. Bifurcation of the wavelength of the interface

is obtained computationally using finite difference techniques, with a 90×90 grid size, to solve for the set of governing equations (6,11,12). A flux-corrected transport algorithm is used to resolve the sharp gradient at the interface.

### Discussion and Computational Results

In order to investigate the interplay of geometric versus boundary-layer length scale; we consider fixed injection of energy per unit volume into the system for a reference condition ( $H=L=5$  cm), in which the input frequency is 1 Hz, amplitude of excitation  $mg_0=20$  milli-g. For a specific dilute system with constant  $(\Delta\rho/\bar{\rho})$  and  $\bar{V}$ ,  $Res=0.3375$ . Variation of the parametric space is considered by varying the length of the cavity in the range  $1\text{ cm} \leq H \leq 15\text{ cm}$  keeping the aspect ratio ( $H=L$ ) fixed to 1. Within this context the length scale issue may be seen from the viewpoint on how the fixed input energy is redistributed in the system as the volume increases or decreases. For a fixed steady background acceleration  $ng_0$  ( $ng_0=10^{-6}g_0$ ,  $g_0=980\text{cm/sec}^2$ ),  $AR=5 \times 10^{-5}$ , the range of parameters is shown in Table 1.

Table 1 – Range of Parameters,  $ng_0=10^{-6}g_0$   
 $Res=0.3375$ ,  $AR=5 \times 10^{-5}$ , \* reference condition.

H (cm)	$Gr_m/Re$	$1/Re$
1	8.1	$1.72 \times 10^{-3}$
2	16.2	$4.29 \times 10^{-4}$
3	24.3	$1.91 \times 10^{-4}$
4	32.4	$1.07 \times 10^{-4}$
* 5	40.5	$6.87 \times 10^{-5}$
6	48.6	$4.77 \times 10^{-5}$
7	56.7	$3.51 \times 10^{-5}$
8	64.8	$2.68 \times 10^{-5}$
9	72.9	$2.12 \times 10^{-5}$
10	80.9	$1.71 \times 10^{-5}$
11	89.1	$1.42 \times 10^{-5}$
12	97.1	$1.19 \times 10^{-5}$
13	105.2	$1.02 \times 10^{-5}$
14	113.3	$8.76 \times 10^{-6}$
15	121.4	$7.63 \times 10^{-6}$

### Dynamics of Interface and Flow Field

The damping characteristic of the system will be investigated for the reference condition corresponding to experimental results. According to the reference results,<sup>1</sup> a

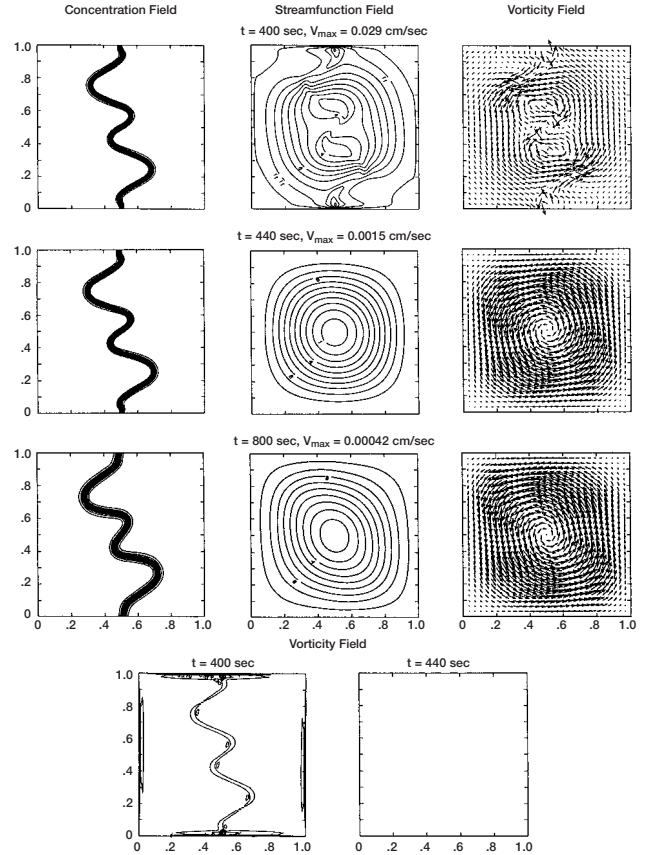


Figure 2.—Damping of quasi-stationary wave due to jump discontinuity in body force for  $t \geq 400$  sec,  $Res = 0.3375$ ,  $AR = 5 \times 10^{-5}$ .

four-mode q-s wave evolves from energy injection into the system ( $H=5\text{cm}$ ,  $Gr_m/Re=40.5$ ). We consider the fate of the four-mode q-s wave for the condition prescribed in equation (2) for  $t_c=400$  sec. Figure 2 shows the damping characteristics of the four-mode q-s wave. The jump discontinuity of energy input into the system causes a global bifurcation of the flow field for which vortex interactions at the interface from the creation term in equation (10) becomes zero. Vortex interaction that causes growth of the q-s wave is annihilated and the flow field decomposes to a weak-rotating flow, which is irrotational as shown by the vorticity field.

The strength of the rotating flow is dictated by the magnitude of the steady background acceleration  $ng_0$ . The remarkable finding is that the interface does not return to its equilibrium position ( $t=440$  sec). The four-mode q-s wave structure remains imbedded into the fluid ( $t=800$  sec). Over a long time scale the q-s wave simply diffuses, since there is no vorticity production, the species continuity equation (5) is decoupled from the flow field. The problem simply reduces to a linear diffusion problem with the four-mode q-s wave as initial condition.



### Damping Characteristics of the Interface

The damping characteristics of the four-mode q-s wave are shown in Figure 3. Since the system is operating near resonance conditions the amplitude of the velocity field varies as a function of time. A subinterval of time ( $390\text{sec} \leq t \leq 410\text{sec}$ ) for the velocity field is shown for both the u and v components. There is an overdamped response in the u component while the v component is critically damped as shown by the undershoot towards steady state. Similar characteristic is shown in the response of the interface. Unlike systems with free surfaces inside bounded containers or drops, there is no underdamped response.

Viscous dissipation for internal q-s waves is very effective at damping motion. These results are similar to the behavior of a damped harmonic oscillator and also

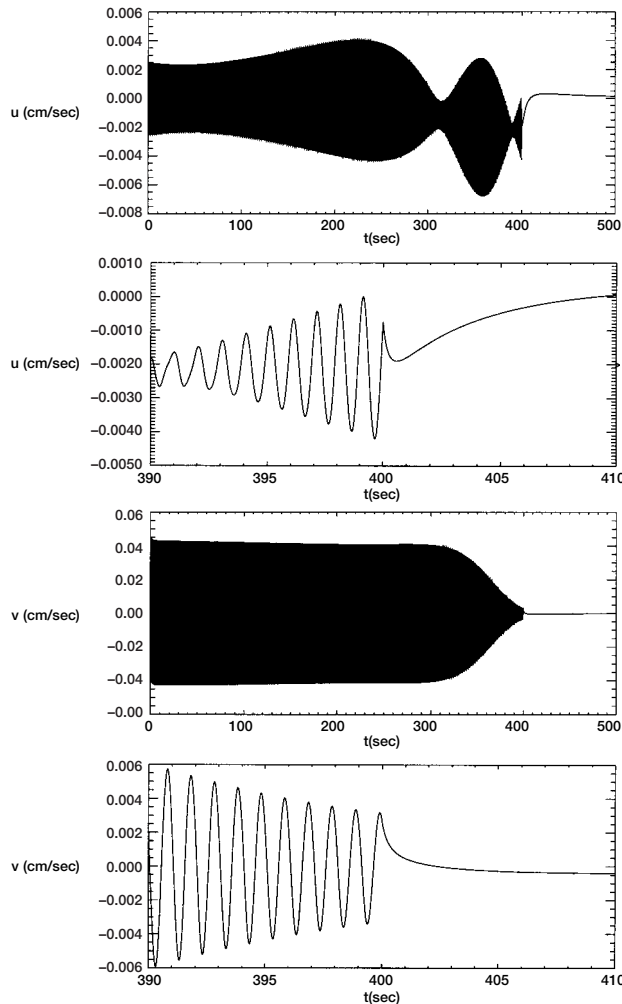


Figure 3.—Time history of velocity field showing overdamped and critically damped response,  $AR = 5 \times 10^{-5}$ ,  $Res = 0.3375$ ,  $f = 1$  Hz, (0.5, 0.5)

occur for certain conditions in the response of a viscous drop.<sup>14,16</sup> Variation of the location (0.11,0.50) in the cavity shows similar trends, Figure 4. The time-asymptotic behavior of the velocity field indicates that a time interval of  $40\text{sec} \leq t \leq 100\text{sec}$  is sufficient for the flow field to reach steady state. The effective damping of the q-s wave can be understood qualitatively from the viewpoint of the decrease of energy of a wave. The energy (E) decreases according<sup>19</sup> to

$$E = c_1 \times e^{-\gamma t} \quad (15)$$

$\gamma$  is the damping coefficient and  $c_1$  is a constant. For a wave,

$$\gamma = 2\bar{v} k^2 \quad (16)$$

$k$  is the wave number defined as  $k = 2\pi / \lambda$ . For the four-mode q-s wave,  $\lambda = 1.33$  cm,  $k = 4.72\text{cm}^{-1}$ ,  $\bar{v} = 0.0108\text{cm}^2/\text{sec}$ , the damping coefficient becomes  $\gamma = 0.48$ . For the wavelength generated by the q-s wave the damping coefficient is significant, thus the energy of the wave decreases quite rapidly.

Further insight is provided from the frequency response of the flow field as shown in Figure 5. Even though there is amplitude variation in the response of the velocity components, the flow field oscillate with the input frequency of 1Hz, shown by the power spectrum  $P_u$

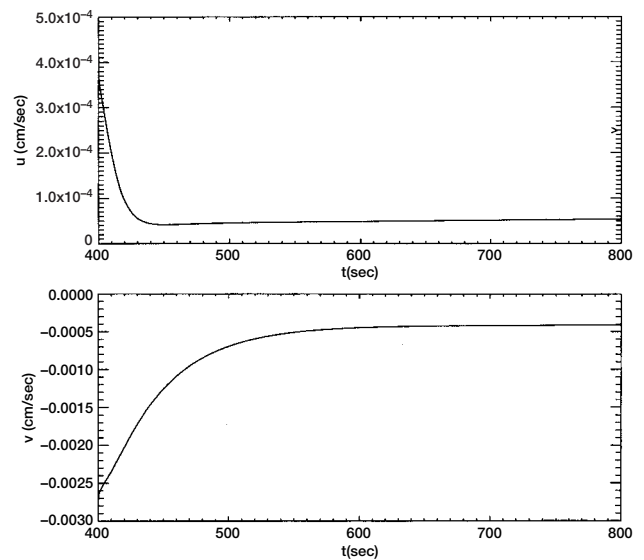


Figure 4.—Time-asymptotic history of velocity field,  $AR = 5 \times 10^{-5}$ ,  $Res = 0.3375$ ,  $f = 1$  Hz, (0.11, 0.50)

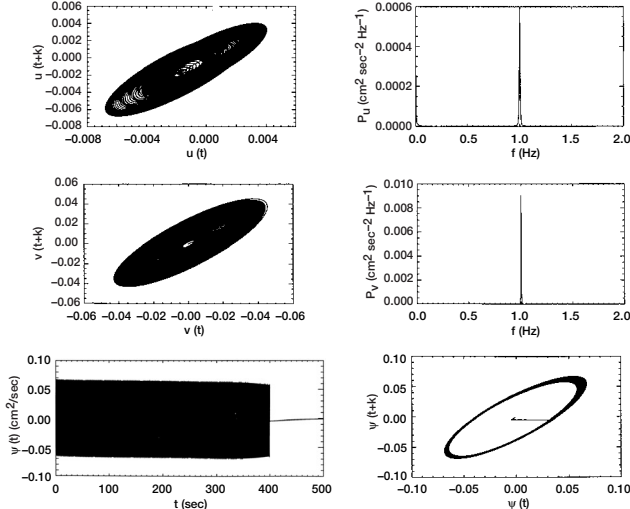


Figure 5.—Response of system to sinusoidal body force,  $Res = 0.3375$ ,  $Ar = 5 \times 10^{-5}$ ,  $f = 1$  Hz, (0.5, 0.5).

and  $P_v$ . For a time interval  $R$ , the power spectrum is defined as,

$$P_v(f) = \frac{1}{R} \left| \int_0^R V(t) e^{-i2\pi f t} dt \right|^2 \quad (17)$$

The effect of the amplitude variation is to produce non-chaotic attractors in phase space as shown by the pseudo-phase space trajectories  $u(t+k)$  and  $v(t+k)$  where  $k$  is a time lag constant. However, the integrated value  $\psi(t)$  shows a nearly constant amplitude variation with a resulting limit cycle attractor.

### Local Bifurcation of the Interface

The local bifurcation of the interface to determine the region of applicability of various length scales, as well as the stability of the four-mode q-s wave is shown in Figure 6. The Stokes-length scale  $\delta$  is applicable when inertia balances viscous effects. This occurs for  $36 < Gr/Re < 65$ , the reference experiment<sup>1</sup> for the stable four-mode q-s wave corresponds to  $Gr_m/Re = 40.5$  ( $H = 5$  cm). The four-mode q-s wave loses stability to two-modes for  $Gr_m/Re < 36$ . As the cavity size increases  $H \rightarrow 8$  cm the wavelength increases for a fixed mode number. The four-mode loses stability for  $Gr > 65$ . In the region  $65 < Gr_m/Re < 102$  the eight-mode is unstable, however it gains stability for  $102 < Gr_m/Re < 122$ . The criterion for the use of the boundary layer scale is based on the parametric range for which the four-mode is stable. In terms of designing a test-cell, this means that  $4.5 \text{ cm} \leq H \leq 8 \text{ cm}$ , for the energy injection to yield the stable four-mode q-s wave.

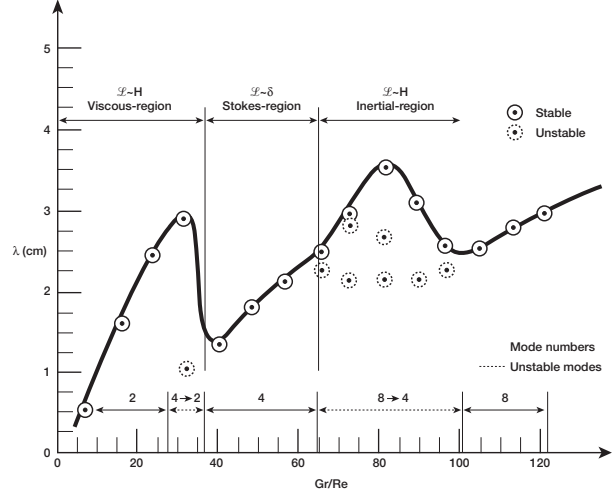


Figure 6.—Bifurcation of mode numbers (wavelength) for fixed input energy as length scale of cavity increases,  $Res = 0.3375$ ,  $Ar = 5 \times 10^{-5}$ .

As the cavity size ( $H < 4.5$  cm) decreases viscous forces become important and the number of modes decrease. On the other hand when the cavity size increases for  $H > 8$  cm ( $Gr_m/Re > 65$ ), inertia dominates and the number of modes increases. The eight-mode gain stability through a supercritical bifurcation in the neighborhood of  $Gr_m/Re = 102$ . The loss of stability from higher to lower mode numbers occurs through a subcritical bifurcation in which the wavelength increases ( $Gr_m/Re = 102$ ,  $Gr_m/Re = 36$ ). In contrast the gain of stability from lower to higher mode number occurs through a supercritical bifurcation. In the inertial region, buoyancy forces overwhelms viscous forces, thus energy injected into the system is much more effectively transmitted against viscous dissipation.

The effect of length scale on the magnitude of the velocity field is shown in Figure 7. The magnitude of the velocity decreases in the Stokes-region. This trend can be understood from the characteristic velocity scale. Since ( $U_c = \Delta\rho / \bar{\rho} \cdot mg_o / \bar{v} \cdot \mathcal{L}^2$ ), as the length scale ( $\mathcal{L}$ ) transition from geometry ( $H$ ) to boundary layer ( $\delta = \sqrt{\bar{v} / \omega}$ ) in which ( $H \gg \delta$ ), there is a decrease in the characteristic velocity. The velocity reaches a minimum in the transition region from four to eight modes, and increases for  $Gr_m/Re > 105$  in the inertial region.

### Effect of Background Acceleration

For a fixed energy input as well a geometrical length, the effect of the background acceleration ( $ng_o$ ) on the stability of the four-mode q-s wave is investigated (Figure 8). Note that the parametric space ( $Gr_m/Re = 41$ )

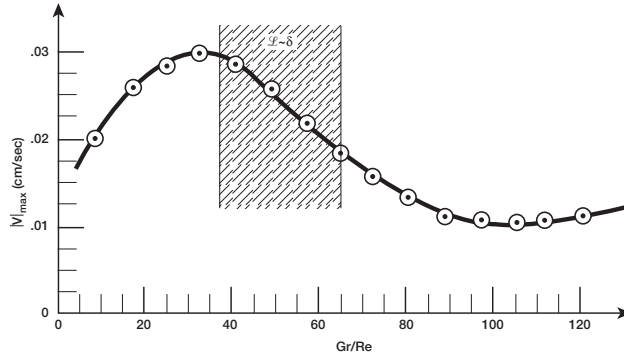


Figure 7.—Magnitude of velocity field as cavity size increases for fixed energy input into system,  $Res = 0.3375$ ,  $Ar = 5 \times 10^{-5}$ .

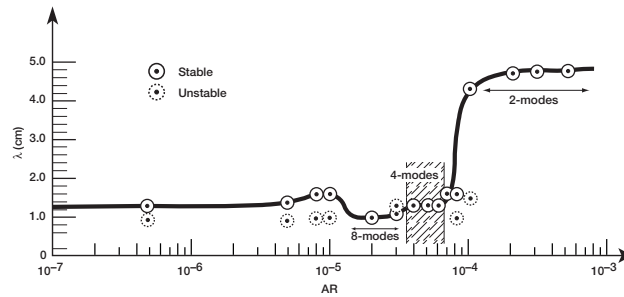


Figure 8.—Stability region of four-mode quasi-stationary wave,  $Res=0.3375$ ,  $Gr/Re=41$ .

corresponds to the Stokes-region in which the boundary layer scale applies ( $\mathcal{L}=\delta$ ). For the reference condition of  $1\mu g$  ( $AR=5 \times 10^{-5}$ ), the region of stability of the four-mode q-s wave is  $3 \times 10^{-5} < AR < 7 \times 10^{-5}$  or in terms of g-level ( $6.0 \times 10^{-7} g_0 < ng_0 < 1.4 \times 10^{-6} g_0$ ). For  $AR > 7 \times 10^{-5}$ , the intensity of the background mean flow destabilizes the higher order modes. The four-mode q-s wave undergoes a supercritical bifurcation to two modes. Beyond  $AR \approx 10^{-3}$  q-s waves no longer exist, buoyancy effects dominate in the limit and the interface overturns to a stably stratified configuration.

The existence of q-s waves occurs for  $AR \leq 5 \times 10^{-4}$ . As the level of  $ng_0$  decreases, there exists a region  $1.2 \times 10^{-5} < AR < 3 \times 10^{-5}$ , for which the eighth-mode surprisingly becomes stable, this corresponds to  $2.4 \times 10^{-7} g_0 < ng_0 < 6.0 \times 10^{-7} g_0$ . In this region the background flow field becomes weaker thus resulting in an increase in the number of modes. However, for  $AR < 1.2 \times 10^{-5}$  the eighth-mode loses stability. This means that the weak flow field created by the steady component of the body force ( $ng_0$ ) is important in terms of stabilizing the eight mode q-s wave. Further decrease of the g-level for  $AR < 1 \times 10^{-7}$  ( $ng_0 < 2 \times 10^{-9} g_0$ ) is no longer effective. This shows that a finite g-level of the background acceleration is necessary for stabilizing q-s waves, it is not necessary to have absolute zero.

## Summary and Conclusions

For a parametric space in which a four-mode q-s wave exists, when the injected energy into the system suddenly ceases by a jump discontinuity, the damping characteristic of the q-s wave is either overdamped or critically damped as the flow field approaches steady state. The characteristic damping time is on the order of 60 sec. The stability of the four-mode q-s wave is dependent on both the length scale and the background acceleration. The four mode q-s wave is stable for  $4.5 \text{ cm} \leq H \leq 8 \text{ cm}$ , and g-level in the range of  $6.0 \times 10^{-7} g_0 < ng_0 < 1.4 \times 10^{-6} g_0$ .

The dependence on length scale shows that the Stokes-region is bounded by the viscous and inertial region. The four-mode q-s wave loses stability supercritically to the eight-mode in the inertial region and subcritically to the second-mode in the viscous region. The reverse bifurcation trend occurs for the dependence of g-level on the stability of four-modes. This study shows that the four-mode q-s wave is stable over a narrow parametric space. For a given energy input, the existence of the four-mode q-s wave has both a geometric length scale as well as a background g-level restriction. For the reference experimental configuration, the system is well parametrized by the Stokes-length scale.

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